where

$$\tilde{\nu}=\tilde{\nu}_{r_T}-\text{angle}\,(r'', r_T) \tag{16}$$

with $\tilde{\nu}_{r_T}$ obtained by inverting

$$r_T=\frac{\tilde{a}(1-\tilde{e}^2)}{1+\tilde{e}\cos\tilde{\nu}_{r_T}} \tag{17}$$

Because the pseudo-orbit is Keplerian, the pseudo speed at $\tilde{r}$ is found with the *vis-viva* equation

$$\tilde{s}=[\mu(2/\tilde{r}-1/\tilde{a})]^{1/2} \tag{18}$$

Hence, the pseudo velocity at $\tilde{r}$ is:

$$\tilde{v}=\mathcal{R}\begin{bmatrix}\Delta i\\ \Delta\Omega\\ \Delta\omega+\Delta\nu+\Delta\nu_{\text{bias}}\end{bmatrix}\frac{v}{|v|}\tilde{s} \tag{19}$$

Writing Eq. (3) in pseudo terms, and this can be done since the pseudo motion is Keplerian,

$$\tilde{v}=\tilde{g}^{-1}(r_T-\tilde{f}\tilde{r}) \tag{20}$$

Equating the right-hand sides of Eqs. (19) and (20),

$$\mathcal{R}v(\tilde{s}/\dot{s})=\tilde{g}^{-1}(r_T-\tilde{f}\tilde{r}) \tag{21}$$

Solving for the actual velocity required to intercept r_T,

$$v_{\text{req}}=\mathcal{R}^T(\dot{s}/\tilde{s})\tilde{g}^{-1}(r_T-\tilde{f}\tilde{r}) \tag{22}$$

To recapitulate, the analysis was directed toward finding $\tilde{r}$ in order that Eq. (3) and the equations of Keplerian motion, which are easily computable, could be used to express v_{req} in the simple form of Eq. (22).

The orbital elements a and e can be selected on the basis of achieving a minimum energy arc,[2] or desired re-entry angle, or desired ballistic flight time. The orientation angles can be obtained[3] knowing r and r_T. Expressions for variations of the elements due to J2 are given by Geyling and Westerman.[4]

Example Test Cases and Numerical Results

The previous theory was programmed and the computational sequence is similar to that which a flight computer would require in an explicit guidance mode for upper-stage steering and engine cutoff.

An ephemeris generation program which employs numerical integration with a J2 field was modified to calculate the miss (minimum distance from r_T) of the simulated trajectory given the initial state r, v_{req} and the results are displayed in Table 1. Small miss is indicated for this ordered grid of target positions. Ignoring J2 in test case 7 causes a 24,400 ft miss; however, when including J2 effects the miss is only 235 ft.

Conclusions

An accurate analytic method has been developed to compute the velocity required for intercept for a space vehicle with perturbations present. Time of flight and perturbation effects involved are side calculations in this development; however, if they are not treated as separate calculations in the analysis, then for even short arcs the expressions become very complex.[5]

Most guidance schemes are tied to a reference trajectory which has been generated by numerical integration and hence can achieve mission changes only close to the reference. This development is independent of a reference trajectory which allows for a flexible respecification of mission. Also, it is efficient in terms of computation time because it does not involve numerical integration and this can be critical in inflight operations.

Acknowledgment

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Announcement: 1978 Author and Subject Index

The indexes of the six AIAA archive journals (*AIAA Journal, Journal of Aircraft, Journal of Energy, Journal of Guidance and Control, Journal of Hydronautics, Journal of Spacecraft and Rockets*) will be combined and mailed separately early in 1979. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1978 will be included. Librarians will receive one copy of the index for each subscription which they have. Any AIAA member who subscribes to one or more Journals will receive one index. Additional copies may be purchased by anyone, at $10 per copy, from the Circulation Department, AIAA, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

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